

GlobustVP

Convex Relaxation for Robust Vanishing Point Estimation in Manhattan World

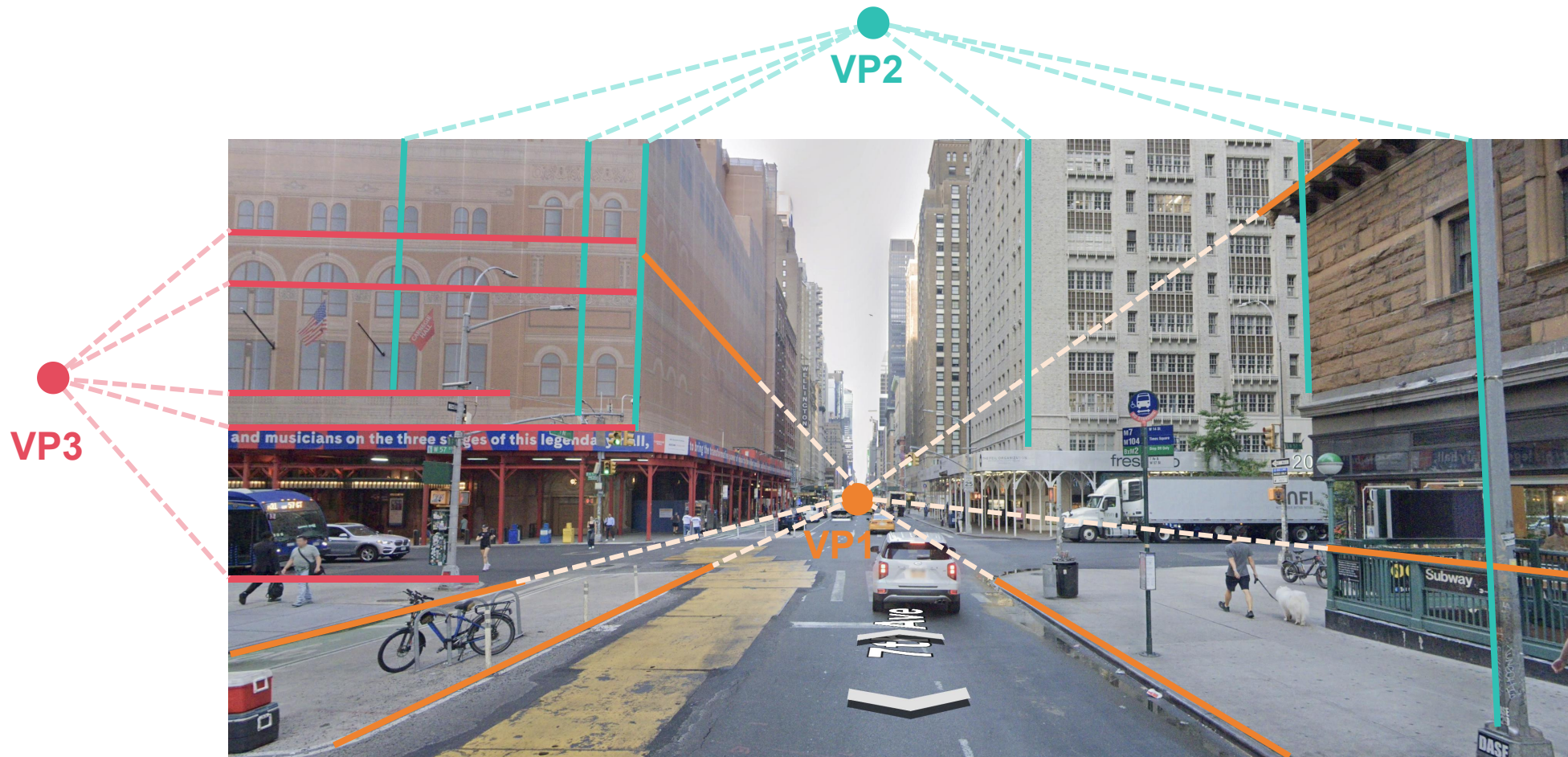
Bangyan Liao^{1,2*}, Zhenjun Zhao^{3*}, Haoang Li⁴, Yi Zhou⁵,
Yingping Zeng⁵, Hao Li¹, Peidong Liu¹

<https://github.com/WU-CVGL/GlobustVP/>



Vanishing Point in Manhattan World

projections of parallel lines intersect at a single point



Applications of Vanishing Point

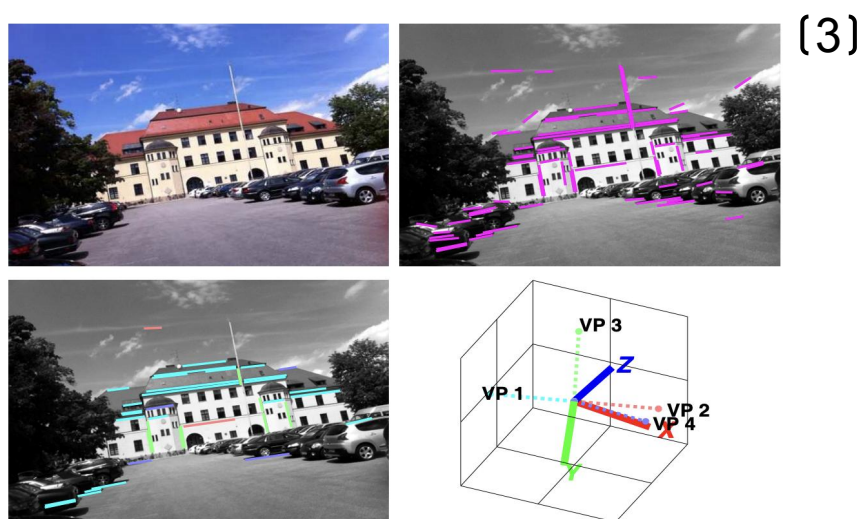
Structure from Motion (SFM)



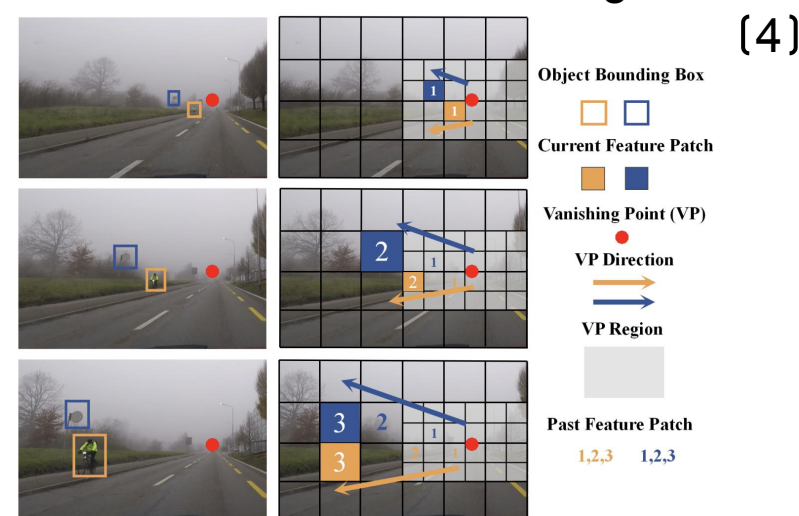
Simultaneous Localization And Mapping (SLAM)



Camera Rotation Estimation

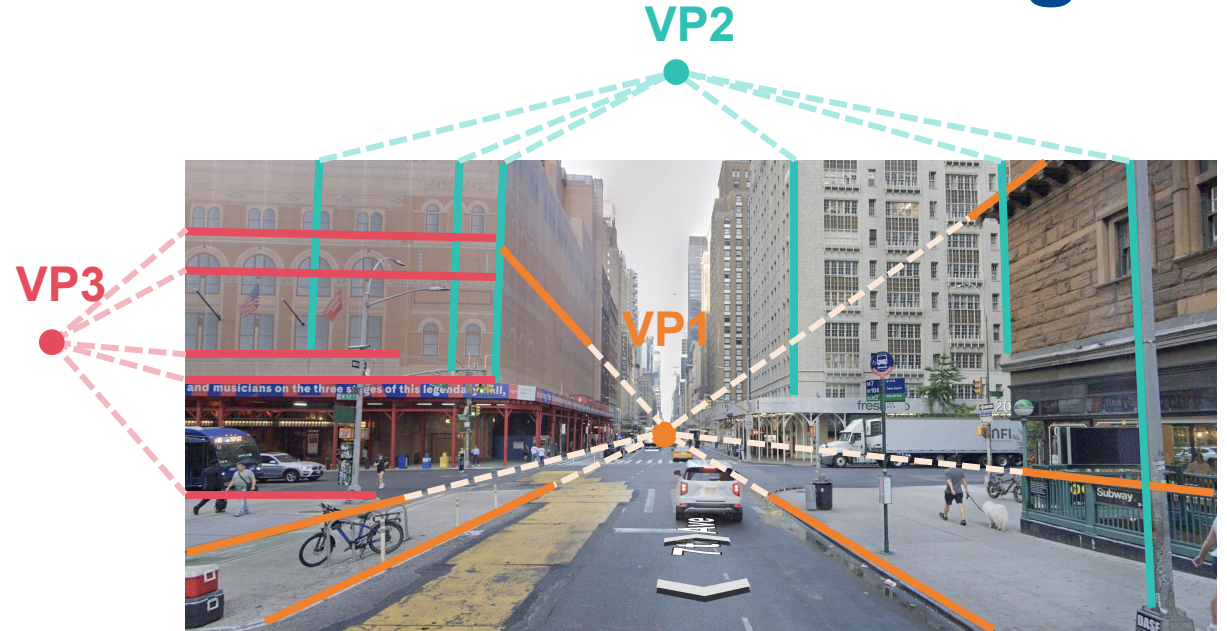


Structural Understanding



- [1] Liu S, Yu Y, Pautrat R, et al. 3d line mapping revisited. CVPR 2023.
 [2] Li Y, Yunus R, Brasch N, et al. RGB-D SLAM with structural regularities. ICRA 2021.
 [3] Lee J K, Yoon K J. Real-time joint estimation of camera orientation and vanishing points. CVPR 2015.
 [4] Guo D, Fan D P, Lu T, et al. Vanishing-point-guided video semantic segmentation of driving scenes. CVPR 2024.

Our Problem: VPs Estimation & Line Labeling



Input: Unlabeled Lines
Intrinsic Parameters



Output: Labeled Lines
Vanishing Points (VP)



**Global
Optimality**



Efficiency



**Outlier
Robustness**



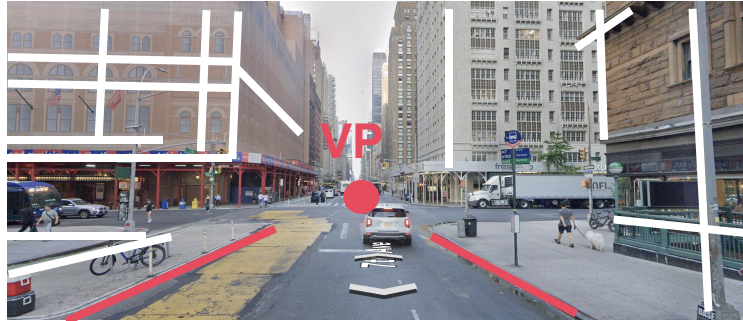
Generalization

Related Works (1/3)

RANdom SAmple Consensus (RANSAC)



Minimal Lines Sampling



VP Hypothesize



Inliers Counting



**Global
Optimality**



Efficiency



**Outlier
Robustness**



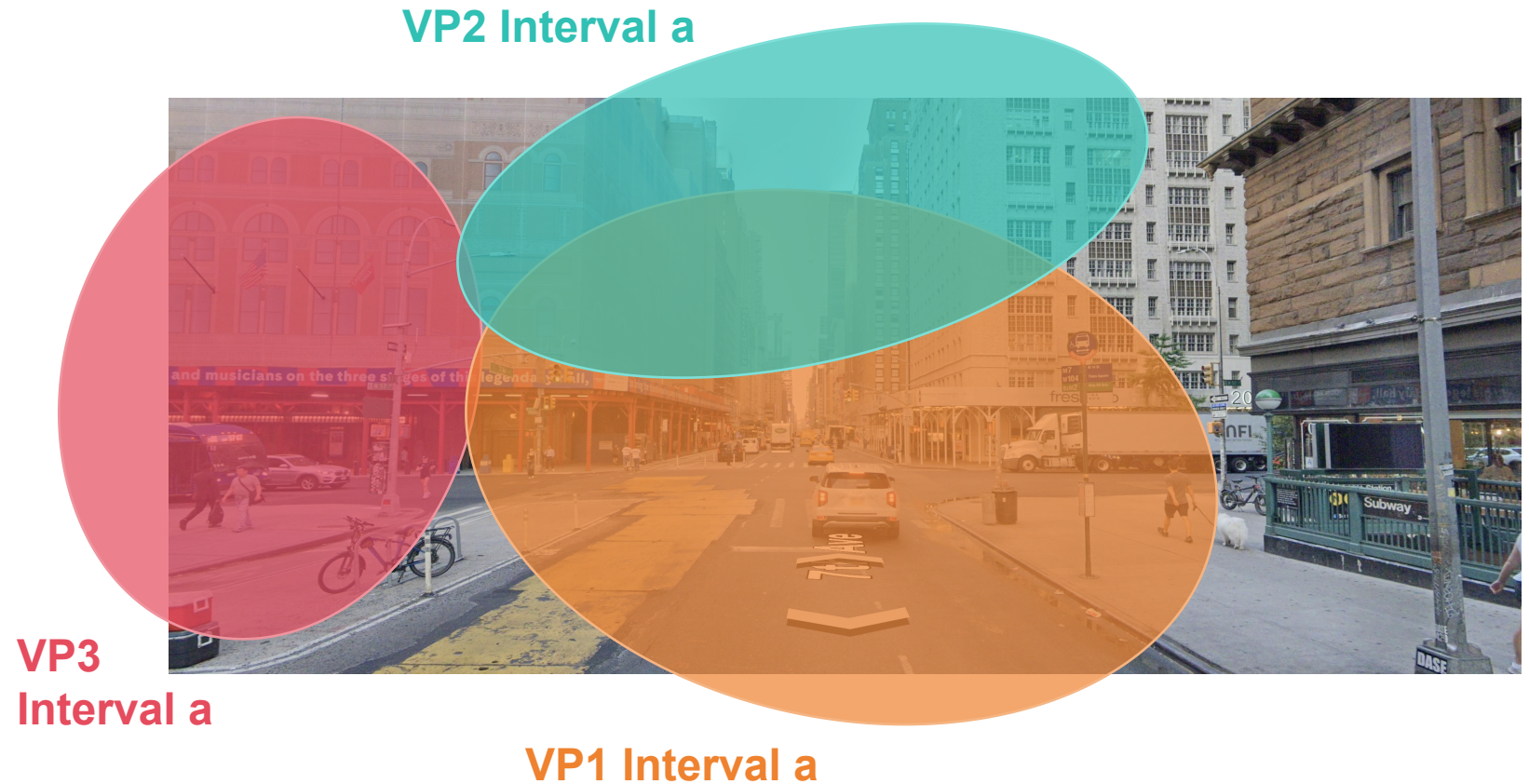
Generalization

- [1] Bazin J C, Pollefeys M. 3-line RANSAC for orthogonal vanishing point detection. IROS 2012.
- [2] Mirzaei F M, Roumeliotis S I. Optimal estimation of vanishing points in a manhattan world. ICCV 2011.
- [3] Sinha S N, Steedly D, Szeliski R, et al. Interactive 3D architectural modeling from unordered photo collections. TOG 2008.
- [4] Tardif J P. Non-iterative approach for fast and accurate vanishing point detection. ICCV 2009.
- [5] Toldo R, Fusiello A. Robust multiple structures estimation with j-linkage. ECCV 2008.
- [6] Zhang L, Lu H, Hu X, et al. Vanishing point estimation and line classification in a manhattan world with a unifying camera model. IJCV 2016.
- [7] Zuliani M, Kenney C S, Manjunath B S. The multiransac algorithm and its application to detect planar homographies. ICIIP 2005.

Related Works (2/3)

Branch and Bound (BnB)

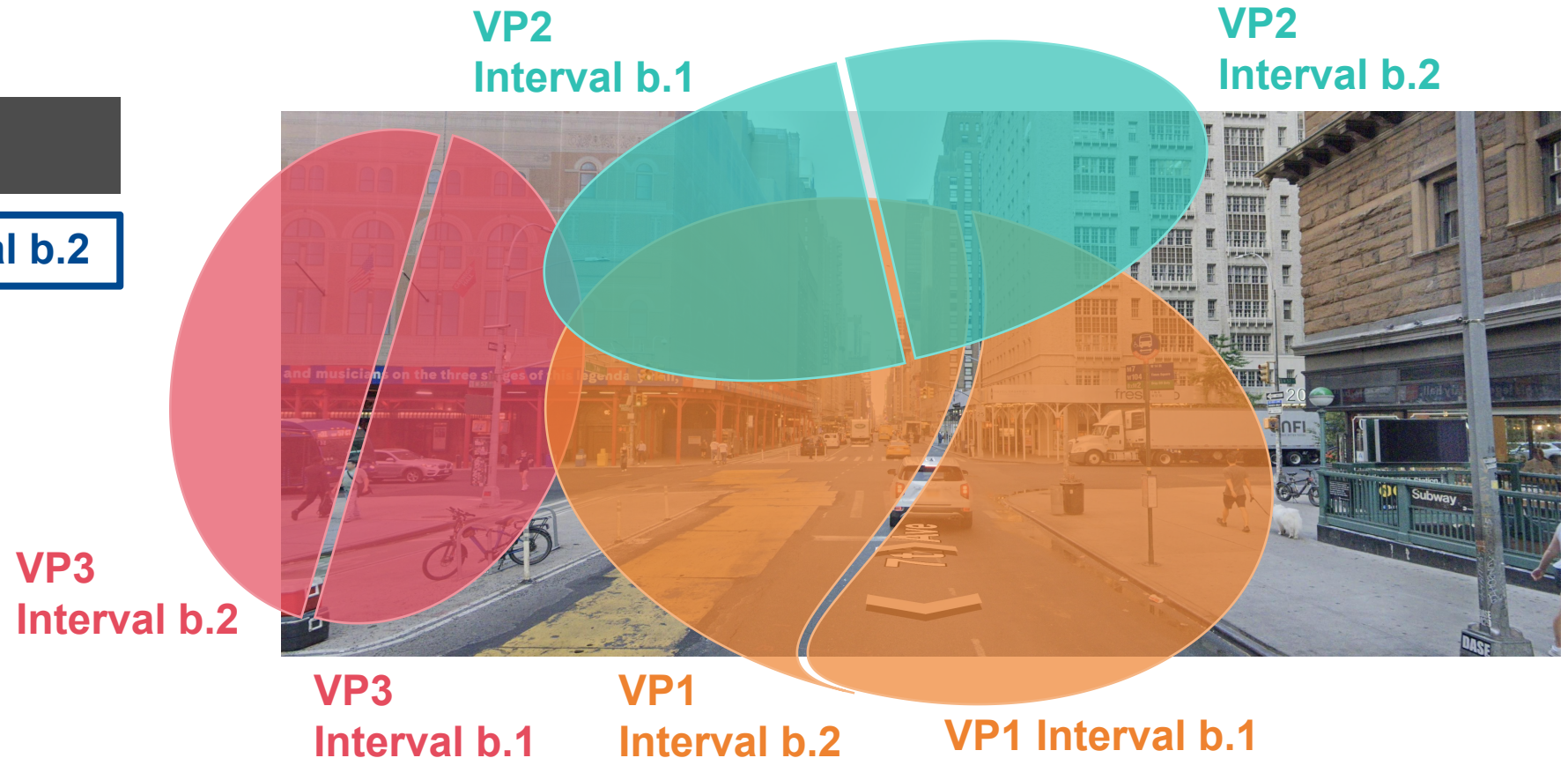
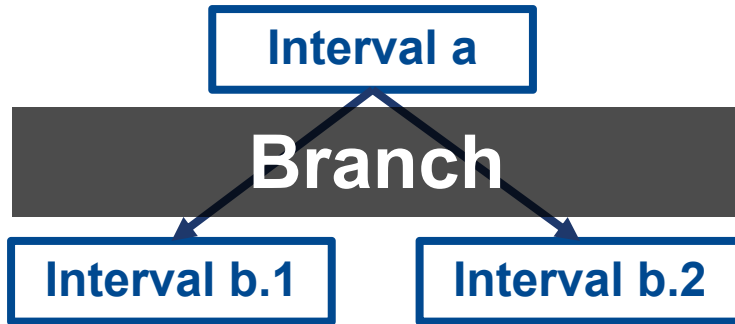
Interval a



- [1] Bazin J C, Seo Y, Demonceaux C, et al. Globally optimal line clustering and vanishing point estimation in manhattan world. CVPR 2012.
- [2] Bazin J C, Seo Y, Pollefeys M. Globally optimal consensus set maximization through rotation search. ACCV 2012.
- [3] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and efficient vanishing point estimation in Manhattan world. ICCV 2019.
- [4] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and near/true real-time vanishing point estimation in Manhattan world. T-PAMI 2020.

Related Works (2/3)

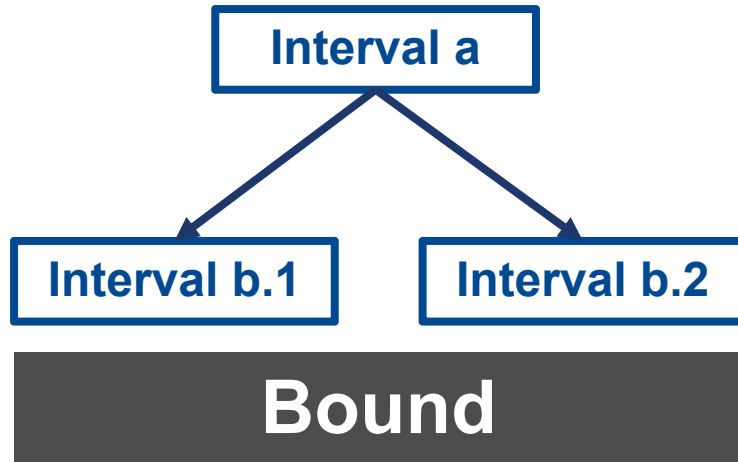
Branch and Bound (BnB)



- [1] Bazin J C, Seo Y, Demonceaux C, et al. Globally optimal line clustering and vanishing point estimation in manhattan world. CVPR 2012.
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Related Works (2/3)

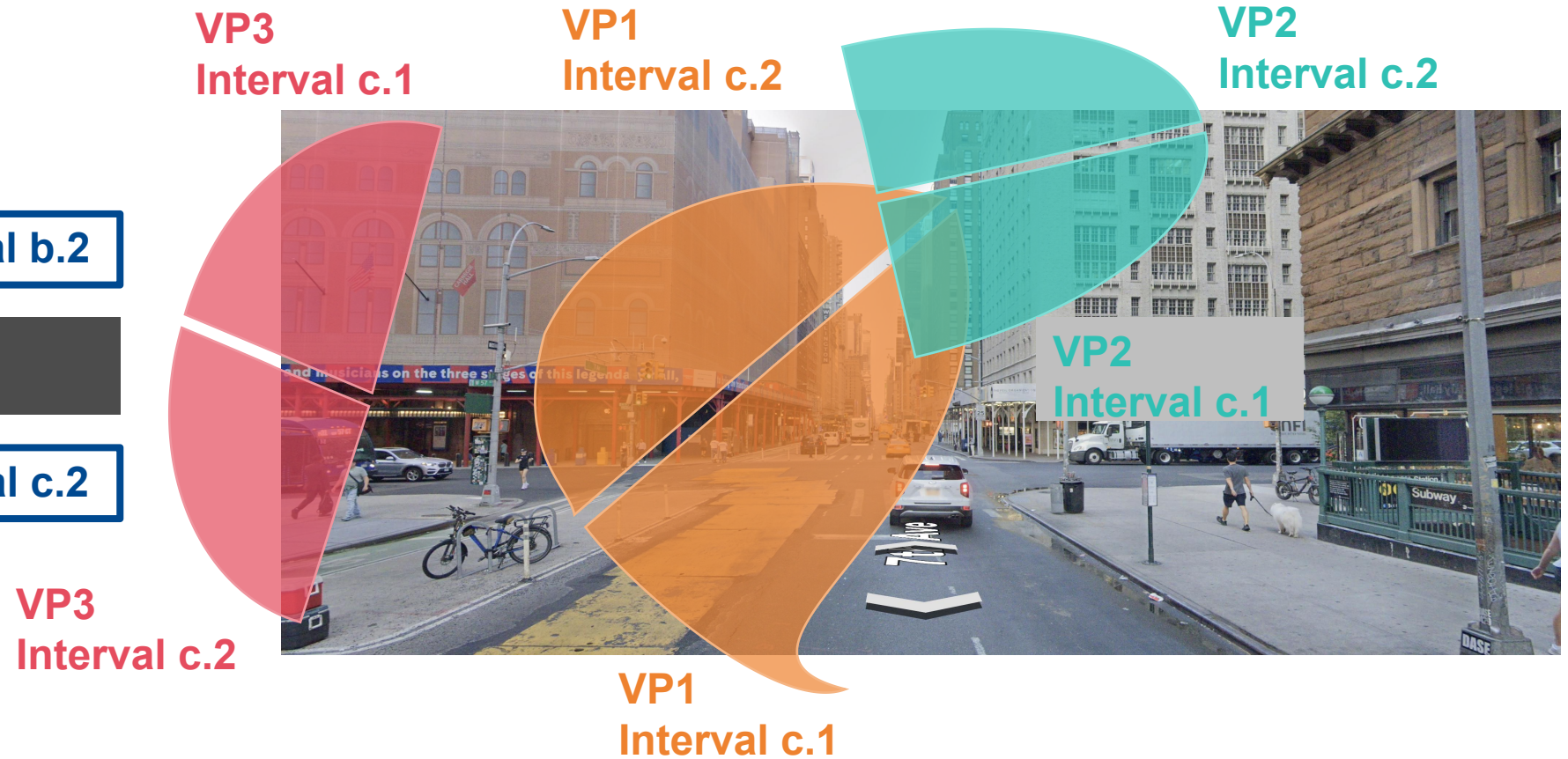
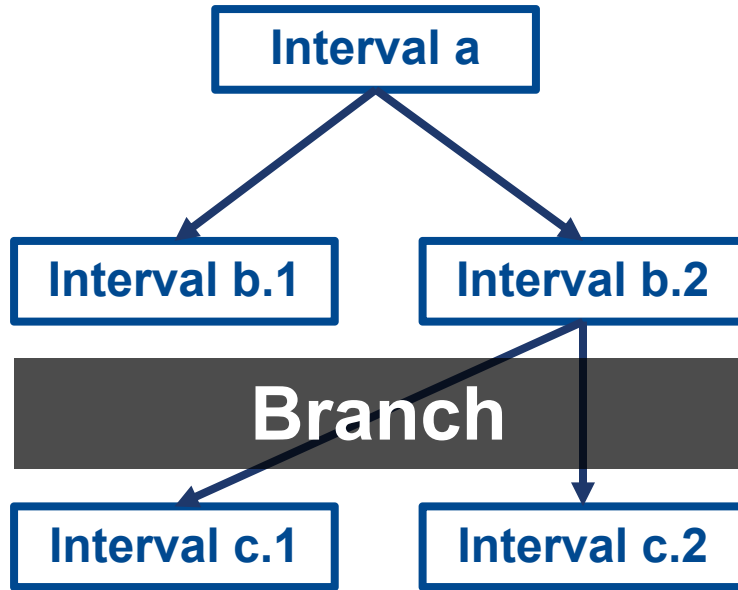
Branch and Bound (BnB)



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Related Works (2/3)

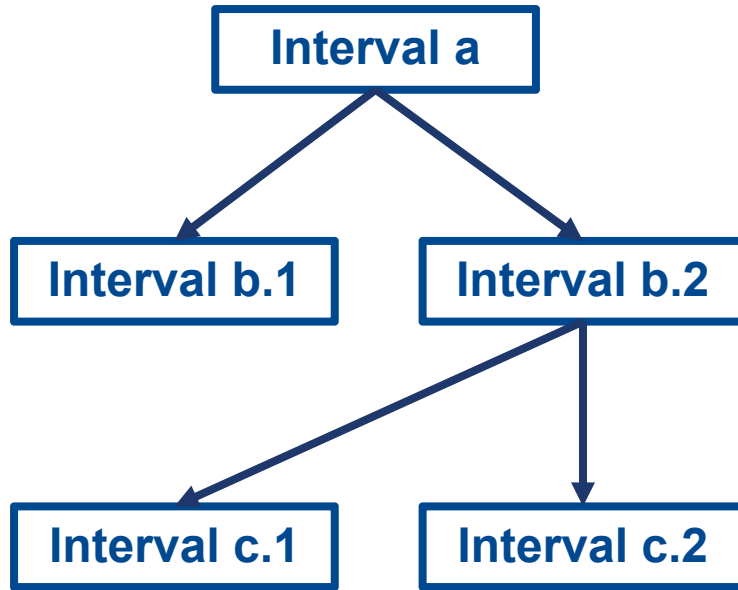
Branch and Bound (BnB)



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- [2] Bazin J C, Seo Y, Pollefeys M. Globally optimal consensus set maximization through rotation search. ACCV 2012.
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Related Works (2/3)

Branch and Bound (BnB)



**Global
Optimality**



Efficiency



**Outlier
Robustness**



Generalization

[1] Bazin J C, Seo Y, Demonceaux C, et al. Globally optimal line clustering and vanishing point estimation in manhattan world. CVPR 2012.

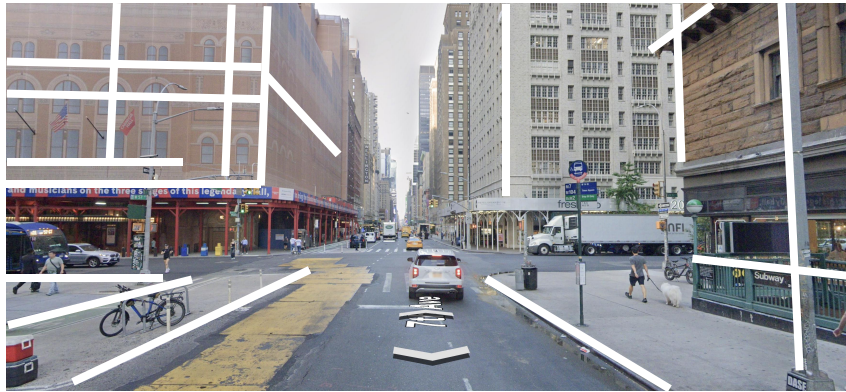
[2] Bazin J C, Seo Y, Pollefeys M. Globally optimal consensus set maximization through rotation search. ACCV 2012.

[3] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and efficient vanishing point estimation in Manhattan world. ICCV 2019.

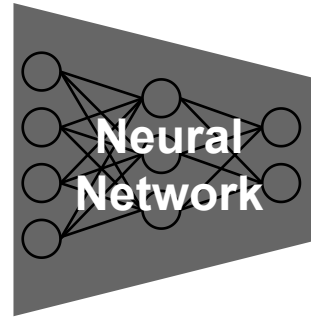
[4] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and near/true real-time vanishing point estimation in Manhattan world. T-PAMI 2020.

Related Works (3/3)

Learning-based



Input: **Raw Image**
Unlabeled Lines (Optional)



VP3



Output: **Vanishing Points**



**Global
Optimality**



Efficiency



**Outlier
Robustness**



Generalization

- [1] Kluger F, Brachmann E, Ackermann H, et al. Consac: Robust multi-model fitting by conditional sample consensus. CVPR 2020.
- [2] Li H, Chen K, Kim P, et al. Learning icosahedral spherical probability map based on bingham mixture model for vanishing point estimation. ICCV 2021.
- [3] Lin Y, Wiersma R, Pintea S L, et al. Deep vanishing point detection: Geometric priors make dataset variations vanish. CVPR 2022.
- [4] Tong X, Ying X, Shi Y, et al. Transformer based line segment classifier with image context for real-time vanishing point detection in Manhattan world. CVPR 2022.
- [5] Zhai M, Workman S, Jacobs N. Detecting vanishing points using global image context in a non-manhattan world. CVPR 2016.
- [6] Zhou Y, Qi H, Huang J, et al. Neurvps: Neural vanishing point scanning via conic convolution. NeurIPS 2019.
- [7] Kluger F, Rosenhahn B. PARSAC: Accelerating robust multi-model fitting with parallel sample consensus. AAAI 2024.

Ours Method: GlobustVP

$$\begin{aligned} \min_{\mathbf{D}, \mathbf{Q}} \quad & \langle [\mathbf{D}\mathbf{N}; c\mathbf{1}_m]^\top, \mathbf{Q} \rangle \\ \text{s.t.} \quad & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{3,*}^\top [\mathbf{D}]_{3,*} = \mathbf{1}, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{2,*} = 0, \\ & [\mathbf{D}]_{2,*}^\top [\mathbf{D}]_{3,*} = 0, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{3,*} = 0, \end{aligned}$$

(Primal Problem)

Insight 1
Problem Reformulation

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq \mathbf{0}, \end{aligned}$$

(Full SDP Problem)

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0,1} = \sum_{i=1}^2 \mathbf{W}_{0,0,i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,j,i} = \mathbf{W}_{0,i,j}, \quad \forall i \in \{1, 2\}, \quad j = 1, \dots, m, \\ & \text{trace}(\mathbf{W}_{0,0,1}) = 1, \quad \mathbf{W}_{0,0,i} = \mathbf{W}_{0,0,2}, \\ & \mathbf{W}_{*,*,i} \succeq \mathbf{0}, \quad \forall i \in \{1, 2\}. \end{aligned}$$

(Single Block SDP Problem)

Insight 2
Convex Relaxation

Insight 3
Iterative Acceleration



**Global
Optimality**



Efficiency



**Outlier
Robustness**



Generalizable

Insight 1: Primal Problem Reformulation

$\min_{\text{VP1, VP2, VP3, Permutation}} \text{Loss}(\text{Lines}, \text{VP1}, \text{VP2}, \text{VP3}, \text{Permutation})$
 How to represent discrete labels?

-> continuous constraint

$$q = q^2 \iff q = 1 \text{ or } q = 0$$

$$q_{\text{VP1}} + q_{\text{VP2}} + q_{\text{VP3}} + q_{\text{Outlier}} = 1$$

-> scales to all lines

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

Permutation Matrix

Column Sum

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

1	1	1	1	1	1
Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

Insight 1: Primal Problem Reformulation

Distance Matrix

VP1	0.3	$2e^{-4}$	0.2	0.6	0.2	0.1
VP2	$2e^{-4}$	0.2	0.3	0.2	$1e^{-4}$	0.2
VP3	0.2	0.1	0.4	0.3	0.3	$1e^{-4}$
Outlier	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6



Line to VP Squared Distance



Maximum Inlier Threshold

VP1	0.3	$2e^{-4}$	0.2	0.6	0.2	0.1
VP2	$2e^{-4}$	0.2	0.3	0.2	$1e^{-4}$	0.2
VP3	0.2	0.1	0.4	0.3	0.3	$1e^{-4}$
Outlier	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6



=

VP1	0.3	$2e^{-4}$	0.2	0.6	0.2	0.1
VP2	$2e^{-4}$	0.2	0.3	0.2	$1e^{-4}$	0.2
VP3	0.2	0.1	0.4	0.3	0.3	$1e^{-4}$
Outlier	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

Insight 1: Primal Problem Reformulation

Primal Problem

$$\min_{\mathbf{D}, \mathbf{Q}} \quad \langle ([\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top])^2, \mathbf{Q} \rangle$$

$$\text{s.t.} \quad \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, \quad (\mathbf{Q})^2 = \mathbf{Q},$$

$$\begin{aligned} [\mathbf{D}]_{1,*} [\mathbf{D}]_{1,*}^\top &= 1, \quad [\mathbf{D}]_{2,*} [\mathbf{D}]_{2,*}^\top = 1, \\ [\mathbf{D}]_{3,*} [\mathbf{D}]_{3,*}^\top &= 1, \quad [\mathbf{D}]_{1,*} [\mathbf{D}]_{2,*}^\top = 0, \\ [\mathbf{D}]_{2,*} [\mathbf{D}]_{3,*}^\top &= 0, \quad [\mathbf{D}]_{1,*} [\mathbf{D}]_{3,*}^\top = 0, \end{aligned}$$

Manhattan Constraint

min
VP1, VP2, VP3
Permutation Matrix

VP1	0.3	2e-4	0.2	0.6	0.2	0.1
VP2	2e-4	0.2	0.3	0.2	1e-4	0.2
VP3	0.2	0.1	0.4	0.3	0.3	1e-4
Outlier	9e-4	9e-4	9e-4	9e-4	9e-4	9e-4
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

Column Sum

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

1	1	1	1	1	1
Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

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VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

Our Method: GlobustVP

$$\begin{aligned} \min_{\mathbf{D}, \mathbf{Q}} \quad & \langle [\mathbf{D}\mathbf{N}; c\mathbf{1}_m]^\top, \mathbf{Q} \rangle \\ \text{s.t.} \quad & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{3,*}^\top [\mathbf{D}]_{3,*} = 1, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{2,*} = 0, \\ & [\mathbf{D}]_{2,*}^\top [\mathbf{D}]_{3,*} = 0, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{3,*} = 0, \end{aligned}$$

(Primal Problem)

1) Primal Problem

Insight 1
Problem Reformulation



Generalizable

Insight 2: QCQP -> Convex Relaxation -> SDP

$$\begin{aligned}
 \min_{\mathbf{D}, \mathbf{Q}} \quad & \langle [\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top]^2, \mathbf{Q} \rangle \\
 \text{s.t.} \quad & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\
 & [\mathbf{D}]_{3,*} [\mathbf{D}]_{3,*}^\top = 1, [\mathbf{D}]_{1,*} [\mathbf{D}]_{2,*}^\top = 0, \\
 & [\mathbf{D}]_{2,*} [\mathbf{D}]_{3,*}^\top = 0, [\mathbf{D}]_{1,*} [\mathbf{D}]_{3,*}^\top = 0,
 \end{aligned}$$

1) Primal Problem
(Primal Problem)

$$\begin{aligned}
 \min_{\omega} \quad & \omega^\top \mathbf{C} \omega \\
 \text{s.t.} \quad & \omega_0 \omega_0^\top = \sum_{i=1}^4 \omega_i \omega_i^\top, \quad j = 1, \dots, m, \\
 & \{\omega_0\}_1^\top \{\omega_0\}_1 = 1, \quad \{\omega_0\}_2^\top \{\omega_0\}_2 = 1, \\
 & \{\omega_0\}_3^\top \{\omega_0\}_3 = 1, \quad \{\omega_0\}_1^\top \{\omega_0\}_2 = 0, \\
 & \{\omega_0\}_1^\top \{\omega_0\}_3 = 0, \quad \{\omega_0\}_2^\top \{\omega_0\}_3 = 0.
 \end{aligned}$$

Quadratically Constrained Quadratic Program (QCQP) Problem
(Full QCQP Problem)

Equivalent to Higher Dimension QCQP

Insight 2: QCQP $\rightarrow$ Convex Relaxation $\rightarrow$ SDP

**Non-convex
QCQP**

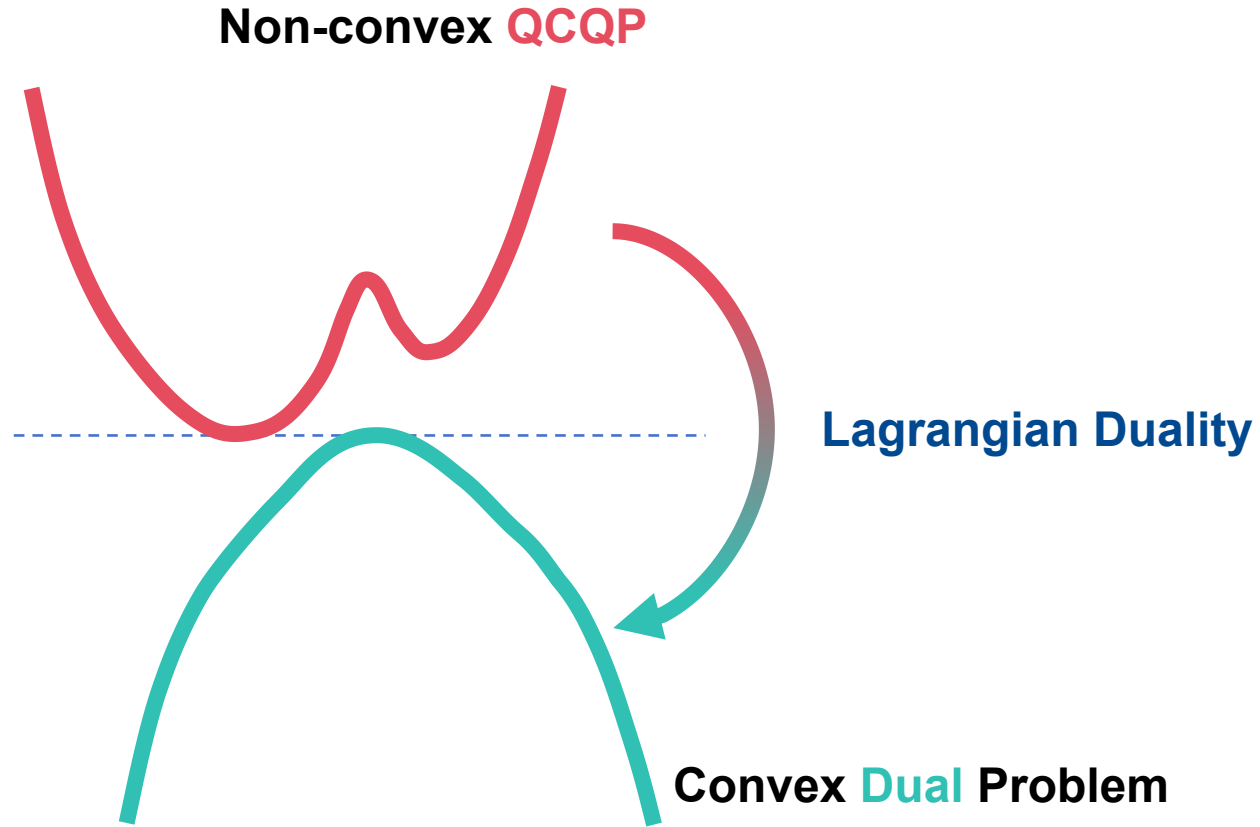
Relax QCQP to Convex Problem

Local Minimum

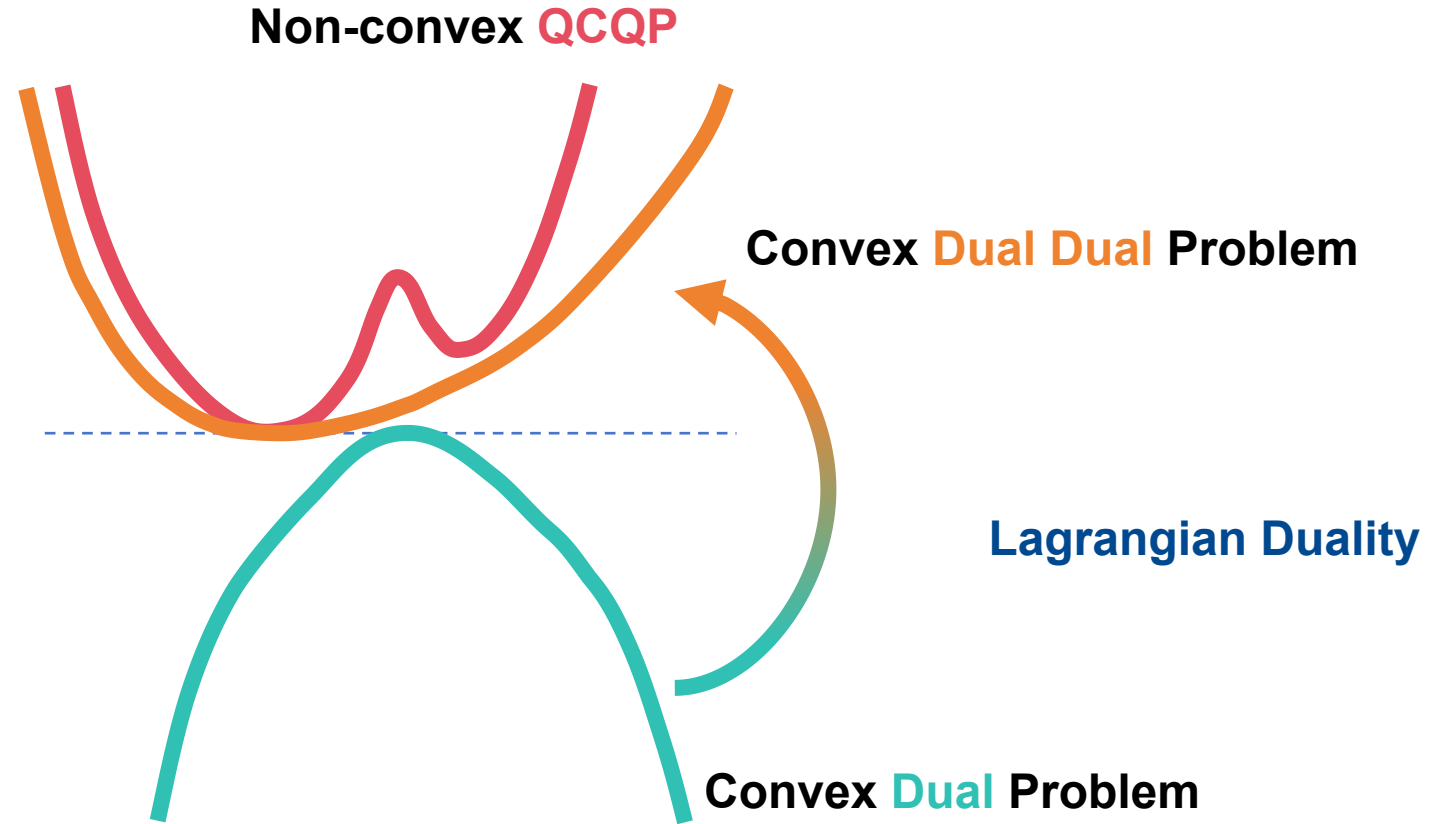
Local Minimum

Global Minimum

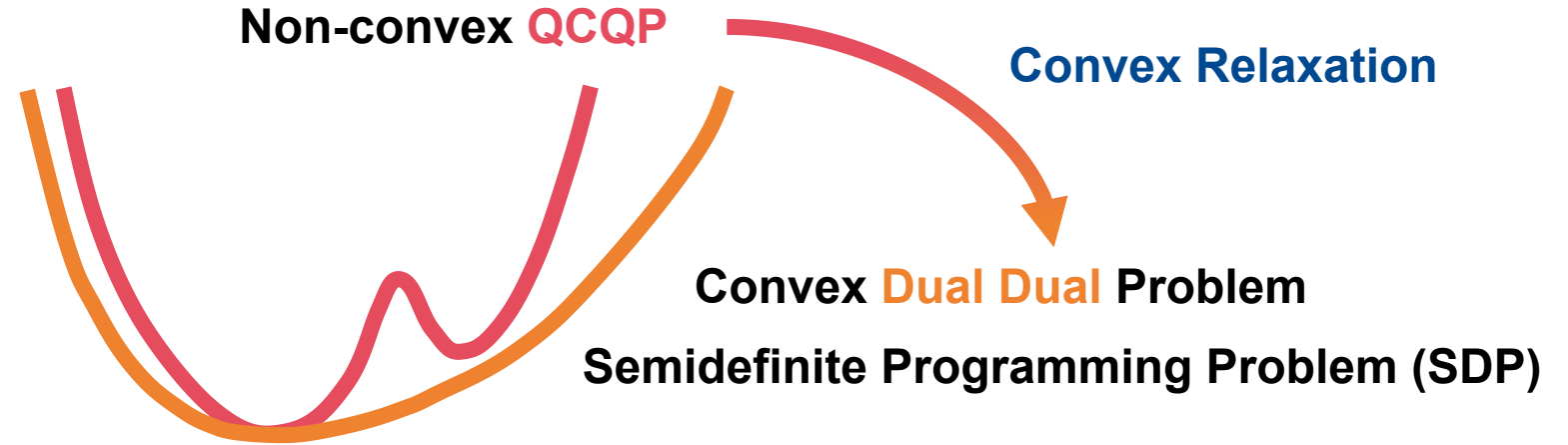
Insight 2: QCQP $\rightarrow$ Convex Relaxation $\rightarrow$ SDP



Insight 2: QCQP $\rightarrow$ Convex Relaxation $\rightarrow$ SDP



Insight 2: QCQP -> Convex Relaxation -> SDP



$$\begin{aligned}
 \min_{\omega} \quad & \omega^\top C \omega \\
 \text{s.t.} \quad & \omega_0 \omega_0^\top = \sum_{i=1}^4 \omega_0 \omega_{i,j}^\top, \quad j = 1, \dots, m, \\
 & \omega_0 \omega_{i,j}^\top = \omega_{i,j} \omega_0, \quad i = 1, \dots, 4, \\
 & \{\omega_0\}_1^\top \{\omega_0\}_1 = 1, \quad \{\omega_0\}_2^\top \{\omega_0\}_2 = 1, \\
 & \{\omega_0\}_3^\top \{\omega_0\}_3 = 1, \quad \{\omega_0\}_1^\top \{\omega_0\}_2 = 0, \\
 & \{\omega_0\}_1^\top \{\omega_0\}_3 = 0, \quad \{\omega_0\}_2^\top \{\omega_0\}_3 = 0.
 \end{aligned}$$

QCQP Problem

(Full QCQP Problem)

equivalent under

=

tight relaxation

$$\begin{aligned}
 \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\
 \text{s.t.} \quad & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\
 & \mathbf{W}_{0,k} = \mathbf{W}_{k,0}^\top, \quad k = 1, \dots, 4, \\
 & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\
 & \mathbf{W} \succeq 0,
 \end{aligned}$$

SDP Problem

(Full SDP Problem)

Ours Method: GlobustVP

$$\begin{aligned} \min_{\mathbf{D}, \mathbf{Q}} \quad & \langle [\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top]^2, \mathbf{Q} \rangle \\ \text{s.t.} \quad & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{3,*}^\top [\mathbf{D}]_{3,*} = 1, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{2,*} = 0, \\ & [\mathbf{D}]_{2,*}^\top [\mathbf{D}]_{3,*} = 0, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{3,*} = 0, \end{aligned}$$

(Primal Problem)

Insight 1
Problem Reformulation

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq \mathbf{0}, \end{aligned}$$

(Full SDP Problem)

Insight 2
Convex Relaxation



**Global
Optimality**



**Outlier
Robustness**



Generalizable

Insight 3: Iterative SDP

$$\min_{\mathbf{W}} \text{trace}(\mathbf{C}\mathbf{W})$$

$$\text{s.t. } \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m,$$

$$\mathbf{W}_{0,j} = \mathbf{W}_{0,j+1}, \quad k = 1, \dots, m,$$

$$\text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\}$$

$$\mathbf{W} \succeq \mathbf{0},$$

(Full SDP Problem)

$$\min_{\mathbf{W}} \text{trace}(\mathbf{C}\mathbf{W})$$

$$\text{s.t. } \mathbf{W}_{0,0,1} = \sum_{i=1}^2 \mathbf{W}_{0,j,i}, \quad j = 1, \dots, m,$$

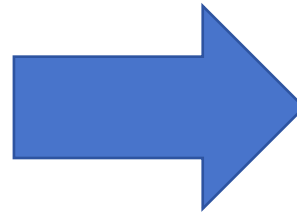
$$\mathbf{W}_{0,j,i} = \mathbf{W}_{0,j,i+1}, \quad i \in \{1, 2\}, \quad j = 1, \dots, m,$$

$$\text{trace}(\mathbf{W}_{0,0,1}) = 1, \quad \mathbf{W}_{0,0,1} = \mathbf{W}_{0,0,2},$$

$$\mathbf{W}_{*,*,i} \succeq \mathbf{0}, \quad \forall i \in \{1, 2\}.$$

(Single Block SDP Problem)

Large SDP problem is not efficient enough



Find VPs one by one!

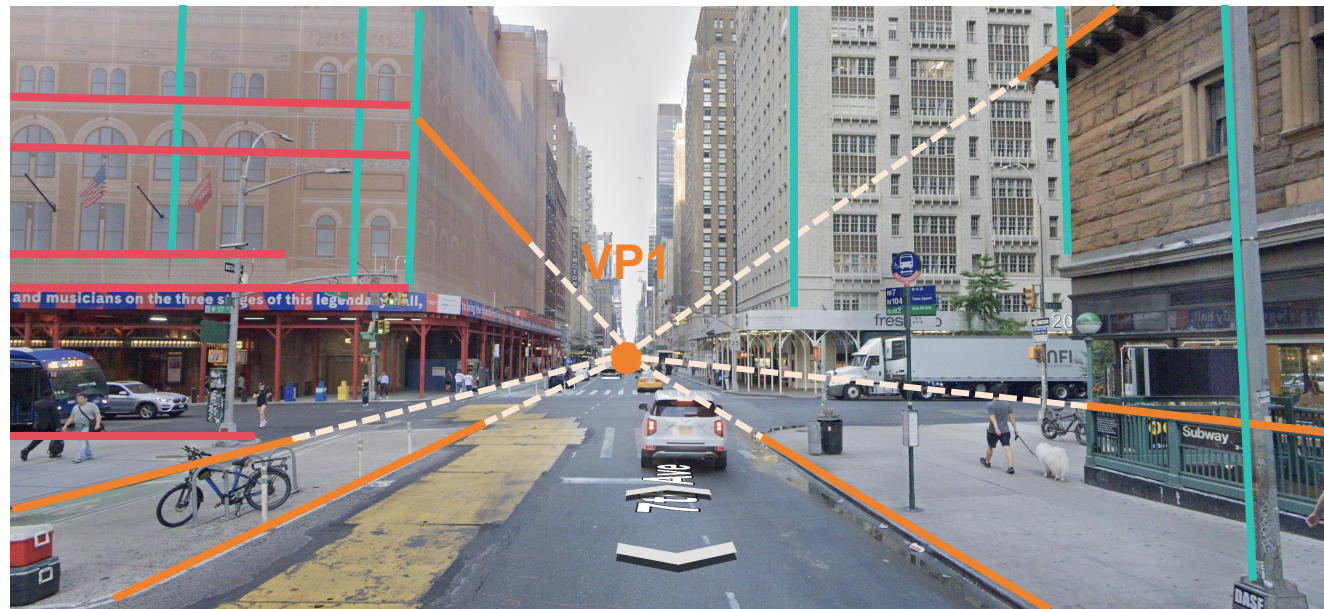
VP1	0.3	2e ⁻⁴	0.2	0.6	0.2	0.1
VP2	2e ⁻⁴	0.2	0.3	0.2	1e ⁻⁴	0.2
VP3	0.2	0.1	0.4	0.3	0.3	1e ⁻⁴
Outlier	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

VP	0.2	2e ⁻⁴	0.4	1e ⁻⁴	0.3	1e ⁻⁴
Outlier	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴	9e ⁻⁴
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

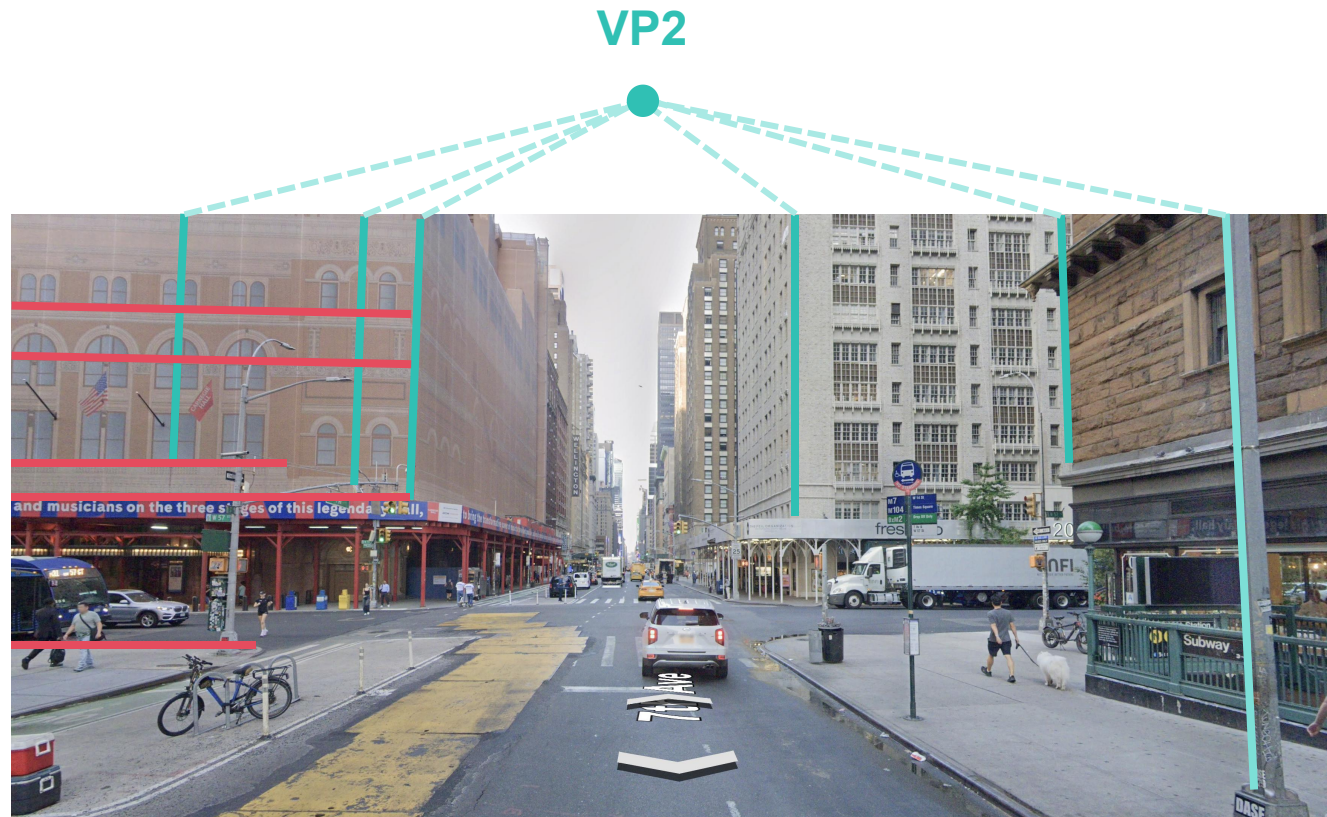
VP	0	1	0	1	0	1
Outlier	1	0	1	0	1	0
	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6

Insight 3: Iterative SDP



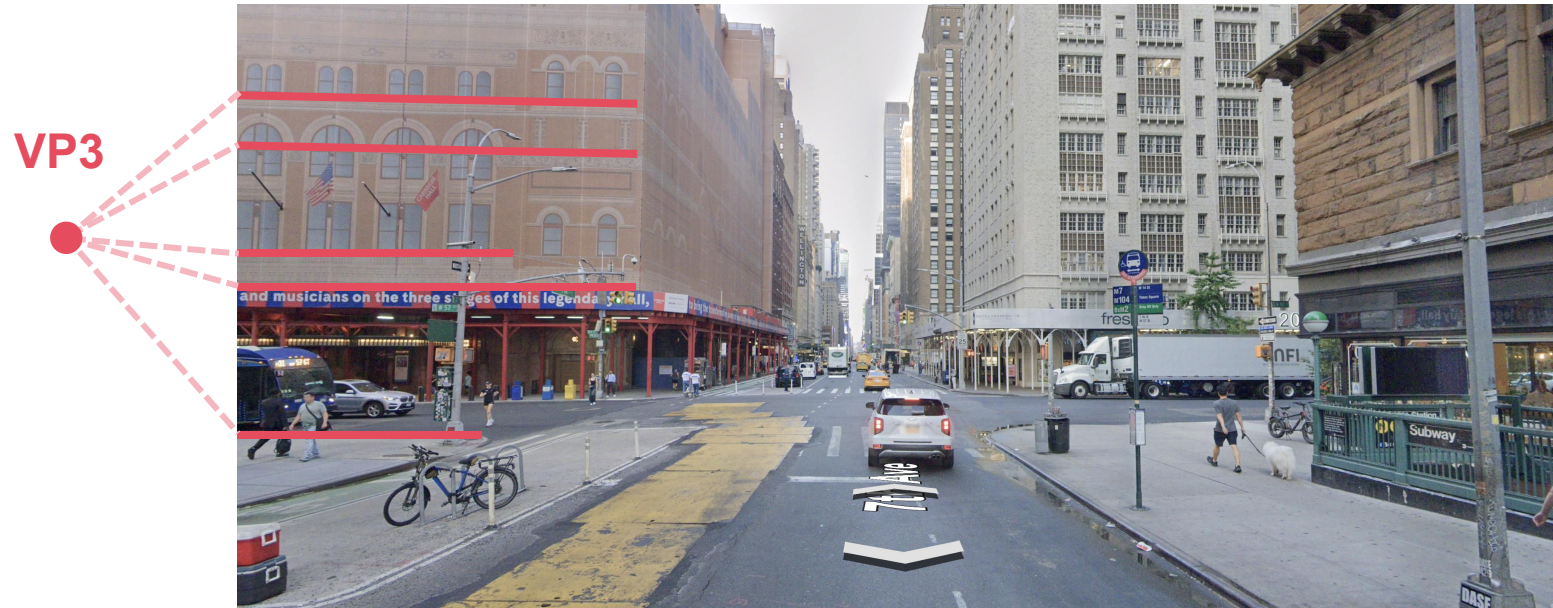
Treat **Line** and **Line** as Outliers, find **VP1**

Insight 3: Iterative SDP



Treat **Line** as Outliers, find **VP2**

Insight 3: Iterative SDP



Find **VP3**

Ours Method: GlobustVP

$$\begin{aligned} \min_{\mathbf{D}, \mathbf{Q}} \quad & \langle [\mathbf{D}\mathbf{N}; c\mathbf{1}_m]^\top, \mathbf{Q} \rangle \\ \text{s.t.} \quad & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{3,*}^\top [\mathbf{D}]_{3,*} = \mathbf{1}, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{2,*} = 0, \\ & [\mathbf{D}]_{2,*}^\top [\mathbf{D}]_{3,*} = 0, [\mathbf{D}]_{1,*}^\top [\mathbf{D}]_{3,*} = 0, \end{aligned}$$

(Primal Problem)

Insight 1
Problem Reformulation

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq \mathbf{0}, \end{aligned}$$

(Full SDP Problem)

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0,1} = \sum_{i=1}^2 \mathbf{W}_{0,0,i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,j,i} = \mathbf{W}_{0,i,j}, \quad \forall i \in \{1, 2\}, \quad j = 1, \dots, m, \\ & \text{trace}(\mathbf{W}_{0,0,1}) = 1, \quad \mathbf{W}_{0,0,i} = \mathbf{W}_{0,0,2}, \\ & \mathbf{W}_{*,*,i} \succeq \mathbf{0}, \quad \forall i \in \{1, 2\}. \end{aligned}$$

(Single Block SDP Problem)

Insight 2
Convex Relaxation

Insight 3
Iterative Acceleration



**Global
Optimality**



Efficiency



**Outlier
Robustness**



Generalizable

Some Qualitative Results

F1-score↑
Consistency error↓



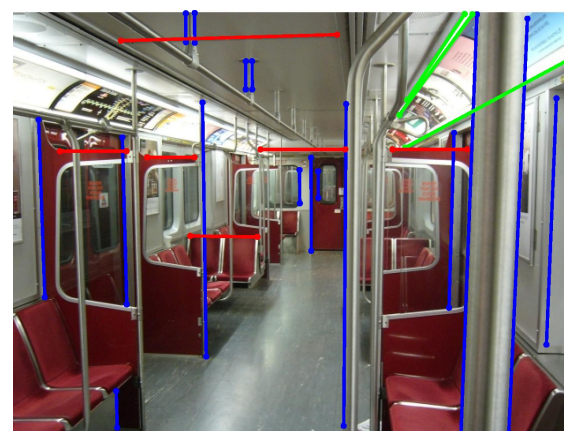
Ground Truth

64.29%, 2.19 pix.



BnB^[1]

85.71%, 1.01 pix.



RANSAC^[2]

100%, 0.41 pix.



Ours

F1-score↑
Consistency error↓



87.60%, 0.52 pix.



BnB^[1]

90.08%, 0.45 pix.



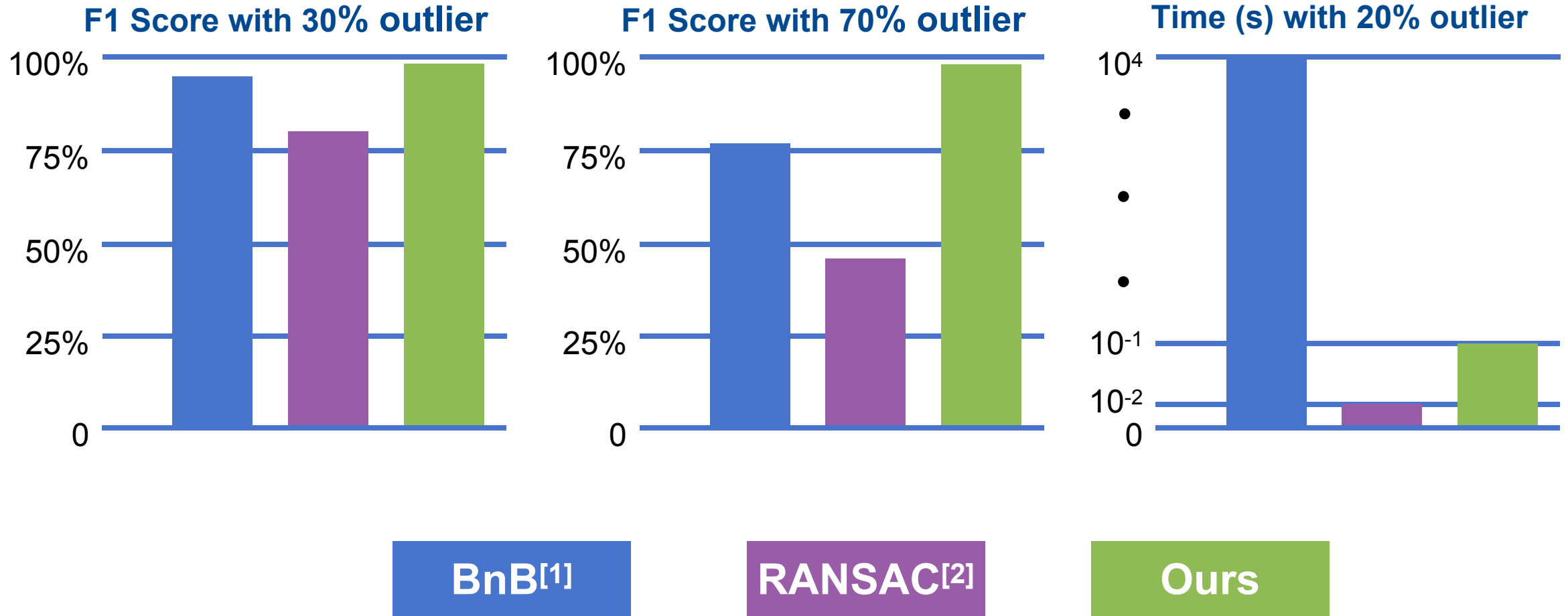
RANSAC^[2]

100%, 0.01 pix.



Ours

Synthetic Comparison

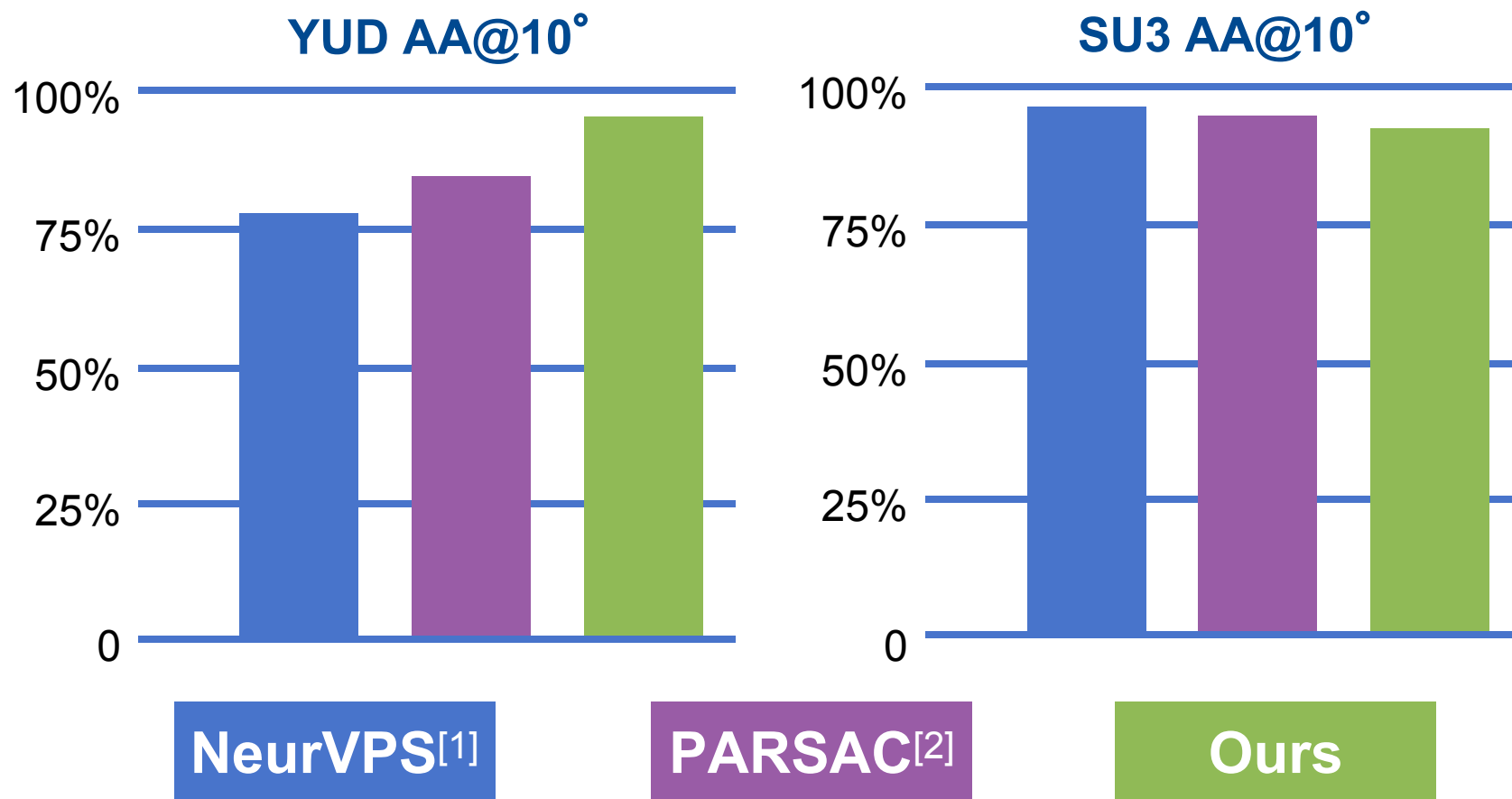


Comparison against Learning-based Method

YUD Dataset



SU3 Dataset



GlobustVP = Globally Optimal Outlier-Robust Vanishing Point Solver



**Global
Optimality**



Efficiency

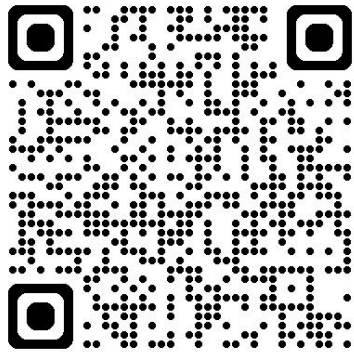


**Outlier
Robustness**



Generalizable

 **GitHub**



<https://github.com/WU-CVGL/GlobustVP/>



See you in Poster Session 4 #102