GlobustVP: Convex Relaxation for Robust Vanishing Point Estimation in Manhattan World

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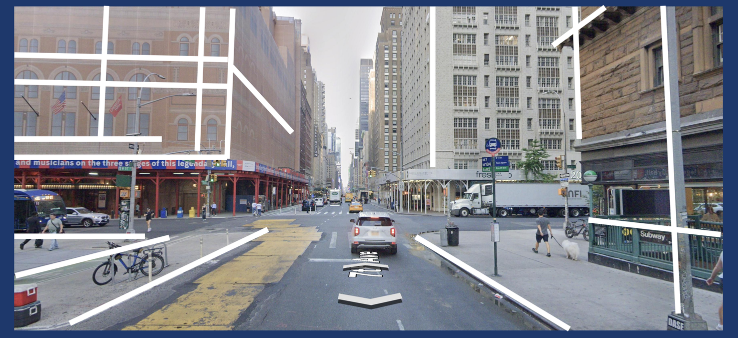
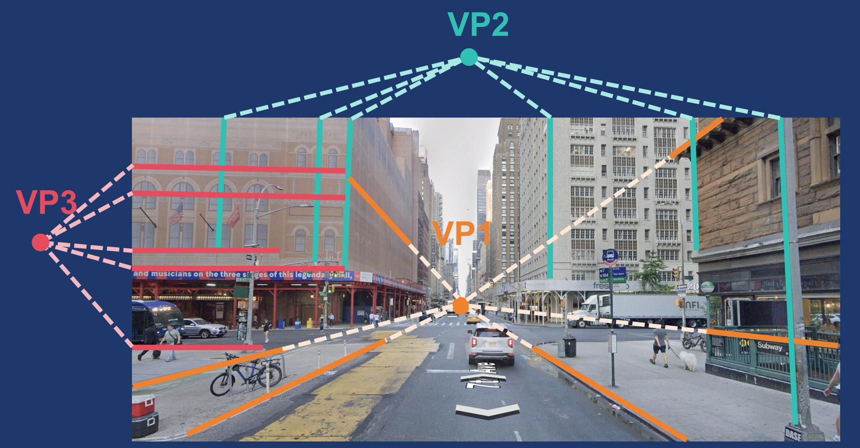
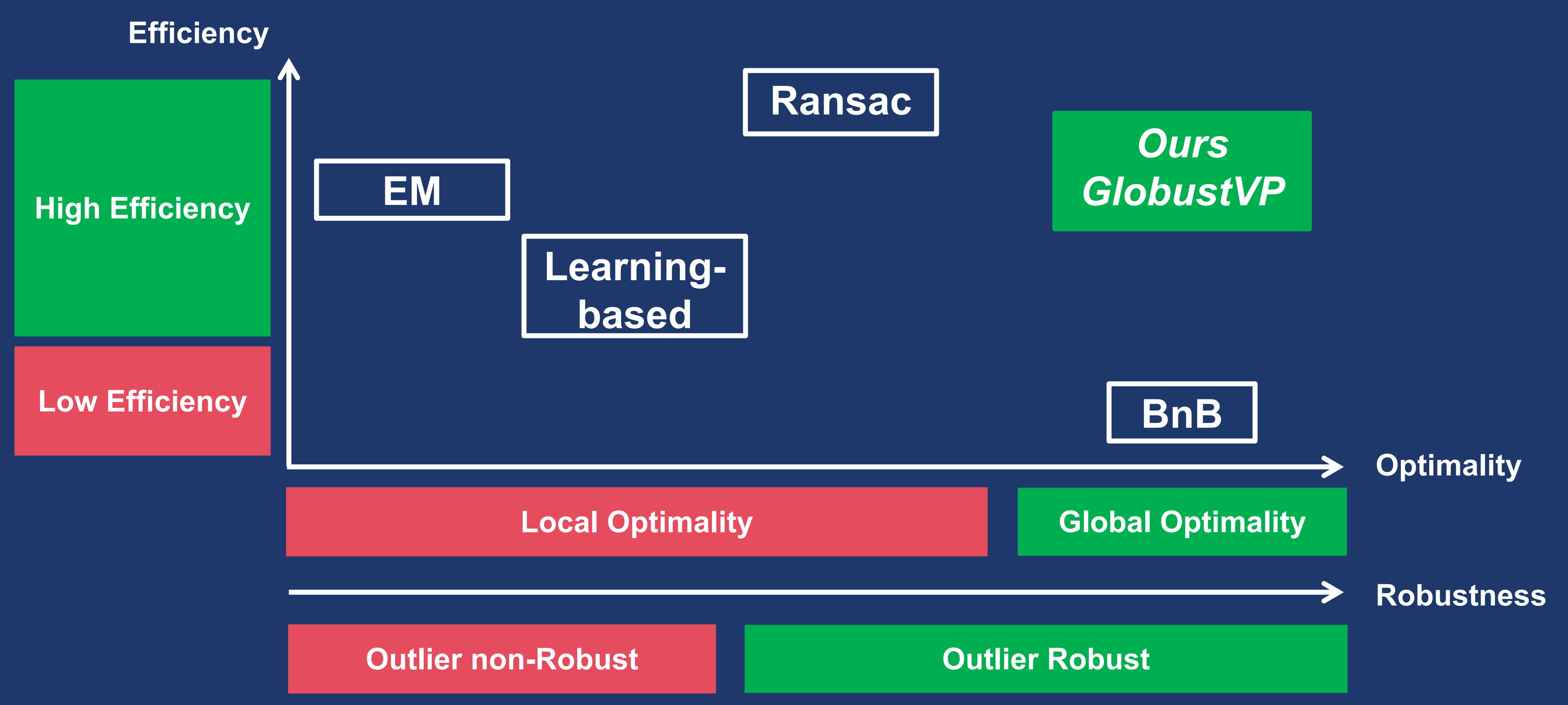
Introduction

■ Task

Input: Unlabeled Lines **Output:** Labeled Lines, Vanishing Points VP1 VP2 VP3

■ Motivation



■ Contribution

- A "soft" association scheme, realized via the proposed truncated multi-selection error, for joint estimation of VPs' location and line-VP association.
- Introducing convex relaxation to reformulate the intermediate QCQP form of the primal problem as a convex SDP problem.
- An iterative solver, GlobustVP, that solves each VP subproblem independently (corresponding to a sub-block of the full SDP problem), achieving global optimality under mild conditions.
- Extensive evaluations on both synthetic and real data demonstrate that our method achieves superior accuracy and robustness, while being on par with prior methods in terms of efficiency.

Convex Relaxation

QCQP

$$\min_{\mathbf{x}\in\mathbb{R}^n} \ \mathbf{x}^\top\mathbf{C}\mathbf{x} \quad \text{s.t.} \ \mathbf{x}^\top\mathbf{A}_i\mathbf{x} = b_i, \quad i=1,\dots,m,$$

Lagrangian Duality →

SDP Dual

$$\max_{\mathbf{y}\in\mathbb{R}^m} \ \mathbf{b}^\top\mathbf{y} \quad \text{s.t.} \ \sum_{i=1}^{m} \mathbf{y}_i\mathbf{A}_i \preceq \mathbf{C}.$$

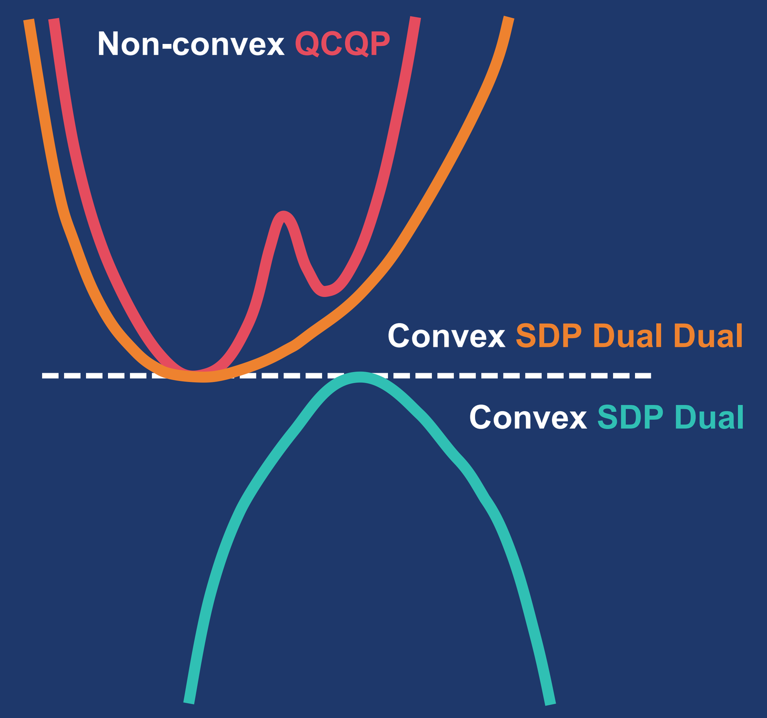
Equivalent ↓ / Convex Relaxation ↘ / Lagrangian Duality ↓

$$\min_{\mathbf{X}\in\mathcal{S}^n} \ \text{trace}(\mathbf{C}\mathbf{X}) \quad \text{s.t.} \ \text{trace}(\mathbf{A}_i\mathbf{X}) = b_i, \quad i=1,\dots,m, \quad \mathbf{X}\succeq\mathbf{0}, \quad \text{rank}(\mathbf{X})=1.$$

Rank Relaxation →

SDP Dual Dual

$$\min_{\mathbf{X}\in\mathcal{S}^n} \ \text{trace}(\mathbf{C}\mathbf{X}) \quad \text{s.t.} \ \text{trace}(\mathbf{A}_i\mathbf{X}) = b_i, \quad i=1,\dots,m, \quad \mathbf{X}\succeq\mathbf{0}.$$



Methods

Primal Problem

$$\min_{\mathbf{D},\mathbf{Q}} \ < ([\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top])^2, \mathbf{Q} >$$
$$\text{s.t.} \ \mathbf{Q}^\top\mathbf{1}_4 = \mathbf{1}_m, \ (\mathbf{Q})^2 = \mathbf{Q},$$
$$[\mathbf{D}]_{1,*}[\mathbf{D}]_{1,*}^\top = 1, \ [\mathbf{D}]_{2,*}[\mathbf{D}]_{2,*}^\top = 1,$$
$$[\mathbf{D}]_{3,*}[\mathbf{D}]_{3,*}^\top = 1, \ [\mathbf{D}]_{1,*}[\mathbf{D}]_{2,*}^\top = 0,$$
$$[\mathbf{D}]_{2,*}[\mathbf{D}]_{3,*}^\top = 0, \ [\mathbf{D}]_{1,*}[\mathbf{D}]_{3,*}^\top = 0,$$

Q

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0

$([\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top])^2$

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
VP1	0.3	$2e^{-4}$	0.2	0.6	0.2	0.1
VP2	$2e^{-4}$	0.2	0.3	0.2	$1e^{-4}$	0.2
VP3	0.2	0.1	0.4	0.3	0.3	$1e^{-4}$
Outlier	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$

From QCQO to SDP →

QCQP Problem

$$\min_{\boldsymbol{\omega}} \ \boldsymbol{\omega}^\top\mathbf{C}\boldsymbol{\omega}$$
$$\text{s.t.} \ \boldsymbol{\omega}_0\boldsymbol{\omega}_0^\top = \sum_{i=1}^{4}\boldsymbol{\omega}_0\boldsymbol{\omega}_{i,j}^\top, \quad j=1,\dots,m,$$
$$\boldsymbol{\omega}_0\boldsymbol{\omega}_{i,j}^\top = \boldsymbol{\omega}_{i,j}\boldsymbol{\omega}_{i,j}^\top, \quad \begin{cases} i=1,\dots,4 \\ j=1,\dots,m \end{cases},$$
$$\{\boldsymbol{\omega}_0\}_1^\top\{\boldsymbol{\omega}_0\}_1 = 1, \quad \{\boldsymbol{\omega}_0\}_2^\top\{\boldsymbol{\omega}_0\}_2 = 1,$$
$$\{\boldsymbol{\omega}_0\}_3^\top\{\boldsymbol{\omega}_0\}_3 = 1, \quad \{\boldsymbol{\omega}_0\}_1^\top\{\boldsymbol{\omega}_0\}_2 = 0,$$
$$\{\boldsymbol{\omega}_0\}_1^\top\{\boldsymbol{\omega}_0\}_3 = 0, \quad \{\boldsymbol{\omega}_0\}_2^\top\{\boldsymbol{\omega}_0\}_3 = 0.$$

Convex Relaxation ↓

SDP Problem

$$\min_{\mathbf{W}} \ \text{trace}(\mathbf{C}\mathbf{W})$$
$$\text{s.t.} \ \mathbf{W}_{0,0} = \sum_{i=1}^{4}\mathbf{W}_{0,4(j-1)+i}, \quad j=1,\dots,m,$$
$$\mathbf{W}_{0,k} = \mathbf{W}_{k,k}, \quad k=1,\dots,4m,$$
$$\text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases} \quad \forall i,j\in\{1,2,3\}$$
$$\mathbf{W}\succeq\mathbf{0},$$

(Full SDP Problem)

← Iterative Solver

Iterative SDP Solver: *GlobustVP*

$$\min_{\mathbf{W}} \ \text{trace}(\mathbf{C}\mathbf{W})$$
$$\text{s.t.} \ \mathbf{W}_{0,0,1} = \sum_{i=1}^{2}\text{trace}(\mathbf{W}_{0,j,i}), \quad j=1,\dots,m,$$
$$\mathbf{W}_{0,j,i} = \mathbf{W}_{j,j,i}, \quad \forall i\in\{1,2\}, \quad j=1,\dots,m,$$
$$\text{trace}(\mathbf{W}_{0,0,1}) = 1, \quad \mathbf{W}_{0,0,1} = \mathbf{W}_{0,0,2},$$
$$\mathbf{W}_{*,*,i}\succeq\mathbf{0}, \quad \forall\, i\in\{1,2\}.$$

Experiment