GlobalPointer: Large-Scale Plane Adjustment with Bi-Convex Relaxation

Bangyan Liao*, Zhenjun Zhao*, Lu Chen, Haoang Li, Daniel Cremers, Peidong Liu

bangyan101.github.io/GlobalPointer

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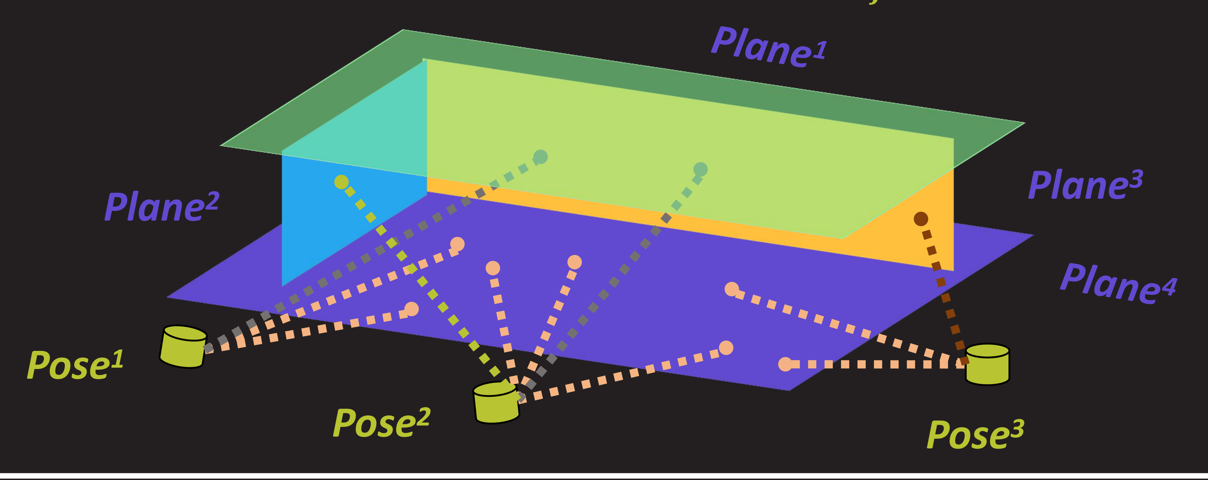
Problem: Plane Adjustment

$$\{\text{Plane}^{1,2,3,4*}, \text{Pose}^{1,2,3*}\} = arg \min_{\text{Plane}^{1,2,3,4}, \text{Pose}^{1,2,3}} \sum_{i=1}^{4}\sum_{j=1}^{3} error(\text{Pose}^j, \text{Plane}^i)$$



Background: Convex Relaxation

QCQP (Primal)

$$\min_{\mathbf{x}\in\mathbb{R}^n} \mathbf{x}^T\mathbf{C}\mathbf{x} \quad s.t.\ \mathbf{x}^T\mathbf{A}_i\mathbf{x} = b_i,\ i = 1,\ldots,m.$$

SDP (Primal)

$$\min_{\mathbf{X}} trace(\mathbf{C}\mathbf{X}) \quad s.t.\ trace(\mathbf{A}_i\mathbf{X}) = b_i,\ \mathbf{X} \succcurlyeq 0,\ i = 1,\ldots,m.$$

SDP (Dual)

$$\min_{\mathbf{y}\in\mathbb{R}^m} \mathbf{b}^T\mathbf{y} \quad s.t.\ \mathbf{C} - \sum_j^n y_j\mathbf{A}_j \succcurlyeq 0,$$

QCQP (Trace Form)

$$\min_{\mathbf{X}} trace(\mathbf{C}\mathbf{X}) \quad s.t.\ trace(\mathbf{A}_i\mathbf{X}) = b_i,\ \mathbf{X} \succcurlyeq 0, rank(\mathbf{X}) = 1,\ i = 1,\ldots,m.$$

Equivalent — Rank Relaxation — Lagrange Dual — Lagrange Dual

The convex relaxation technique serves as a general tool to reformulate the original non-convex QCQP problem into a new convex SDP problem. Although relaxation is applied, researchers have found that strong duality properties still hold for many problems.

Contribution

- A novel optimization strategy: ***Bi-Convex Relaxation*** combines the advantages of both alternating minimization and convex relaxation techniques
- Two algorithmic variants: ***GlobalPointer*** and ***GlobalPointer++*** depend on point-to-plane and plane-to-plane errors, respectively
- Extensive synthetic and real experimental evaluations demonstrate
  - **linear time complexity**
  - **robustness to poor initialization**
  - **similar accuracy as prior methods.**

Methodology 1: Bi-convex Relaxation

- **decouple the original complex formulation into two sub-problems**
- **reformulate each problem using convex relaxation technique**
- **solve each problem alternately until the overall problem converges**

Methodology 2: GlobalPointer

PA (Point-to-Plane Error)

$$\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \sum_{i=1}^{m}\sum_{j=1}^{n} \begin{bmatrix}\mathbf{n}_j\\ d_j\end{bmatrix}^T \mathbf{T}_i\mathcal{B}(i,j)\mathbf{T}_i^T \begin{bmatrix}\mathbf{n}_j\\ d_j\end{bmatrix}$$
$$s.t.\ \mathbf{R}_i \in \mathbf{SO}(3),\ \|\mathbf{n}_j\| = 1$$
$$\mathcal{B}(i,j) = \sum_{k\in obs(i,j)} \begin{bmatrix}\mathbf{P}_k^l\mathbf{P}_k^{l\,T} & \mathbf{P}_k^l\\ \mathbf{P}_k^{l\,T} & 1\end{bmatrix}$$

Pose-Only QCQP

$$\{vec(\mathbf{T})^*\} = arg \min_{vec(\mathbf{T})} \sum_{i=1}^{m} vec(\mathbf{T}_i)^T\mathcal{C}(i)vec(\mathbf{T}_i)$$
$$s.t.\ \mathcal{C}(i) = \sum_{j=1}^{n}(\mathbf{b}_j\mathbf{b}_j^T)\otimes\mathcal{B}(i,j)$$
$$\mathbf{R}_i \in \mathbf{SO}(3).$$

Plane-Only QCQP

$$\{\mathbf{b}^*\} = arg \min_{\mathbf{b}} \sum_{j=1}^{n} \mathbf{b}_j^T\mathcal{D}(j)\mathbf{b}_j$$
$$s.t.\ \mathcal{D}(j) = \sum_{i=1}^{m}\mathbf{T}_i\mathcal{B}(i,j)\mathbf{T}_i^T$$
$$\|\mathbf{n}_j\| = 1.$$

Pose-Only SDP

$$\{\mathbf{X}^*\} = arg \min_{\mathbf{X}} \sum_{i=1}^{m} trace(\mathcal{C}(i)\mathbf{X}_i)$$
$$s.t.\ \mathcal{C}(i) = \sum_{j=1}^{n}(\mathbf{b}_j\mathbf{b}_j^T)\otimes\mathcal{B}(i,j),$$
$$\mathbf{X}_i \succcurlyeq 0,\ \{\text{redundant rotation constraints}\}.$$

Plane-Only SDP

$$\{\mathbf{Y}^*\} = arg \min_{\mathbf{Y}} \sum_{j=1}^{n} trace(\mathcal{D}(j)\mathbf{Y}_j)$$
$$s.t.\ \mathcal{D}(j) = \sum_{i=1}^{m}\mathbf{T}_i\mathcal{B}(i,j)\mathbf{T}_i^T,$$
$$\mathbf{Y}_j \succcurlyeq 0,\ \|\mathbf{n}_j\| = 1.$$

GlobalPointer

Pose-Only SDP → Rounding $\{\mathbf{X}^*\}$ to get pose variables $\mathbf{R}, \mathbf{t}$ → Plane-Only SDP → Rounding $\{\mathbf{Y}^*\}$ to get plane variables $\mathbf{b}$ → Pose-Only SDP

Methodology 3: GlobalPointer++

PA (Plane-to-Plane Error)

$$\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \sum_{i=1}^{m}\sum_{j=1}^{n} \|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\|_2^2 + \left\|\mathbf{n}_{ij}^{l*\,T}\mathbf{R}_i^T\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$s.t.\ \mathbf{R}_i \in \mathbf{SO}(3),\ \|\mathbf{n}_j\| = 1$$
$$(\mathbf{n}_{ij}^{l*}, d_{ij}^{l*}) = arg \min_{\mathbf{n}_{ij}^l, d_{ij}^l} \begin{bmatrix}\mathbf{n}_{ij}^l\\ d_{ij}^l\end{bmatrix}^T \mathcal{B}(i,j) \begin{bmatrix}\mathbf{n}_{ij}^l\\ d_{ij}^l\end{bmatrix}$$

Pose-Only Closed-Form Solver

$$\{\mathbf{R}^*, \mathbf{t}^*\} = arg \min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{m}\sum_{j=1}^{n} \|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\|_2^2 + \left\|\mathbf{n}_{ij}^{l*\,T}\mathbf{R}_i^T\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$= arg \min_{\mathbf{q},\mathbf{t}} \sum_{i=1}^{m}\sum_{j=1}^{n} \mathbf{q}_i^T\mathbf{M}_{ij}\mathbf{q}_i + \left\|\mathbf{n}_{ij}^{l*\,T}\mathbf{R}_i^T\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$s.t.\ \|\mathbf{q}_j\| = 1.$$

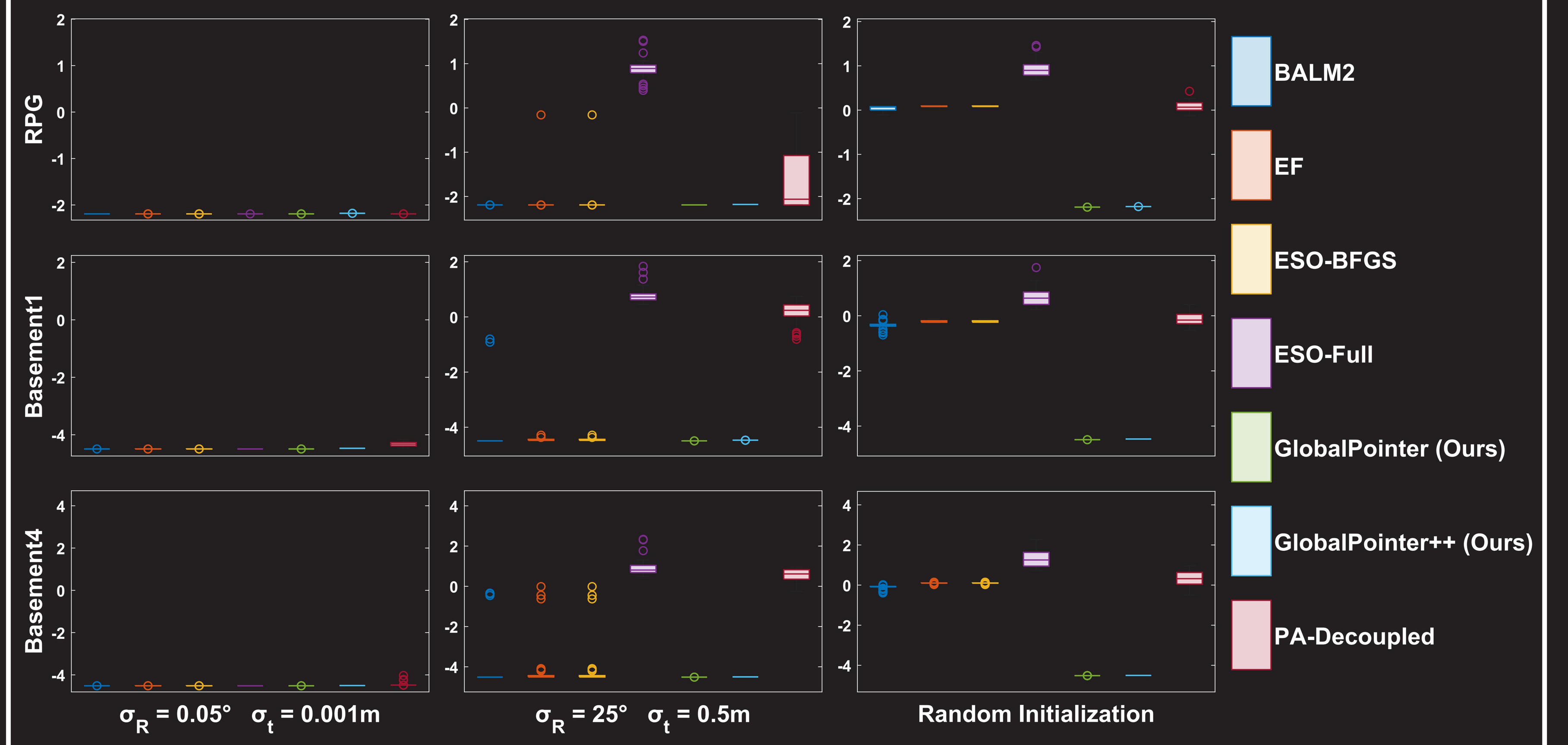
Plane-Only Closed-Form Solver

$$\{\mathbf{n}^*, d^*\} = arg \min_{\mathbf{n},d} \sum_{i=1}^{m}\sum_{j=1}^{n} \|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\|_2^2 + \left\|\mathbf{n}_{ij}^{l*\,T}\mathbf{R}_i^T\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$s.t.\ \|\mathbf{n}_j\| = 1.$$

GlobalPointer++

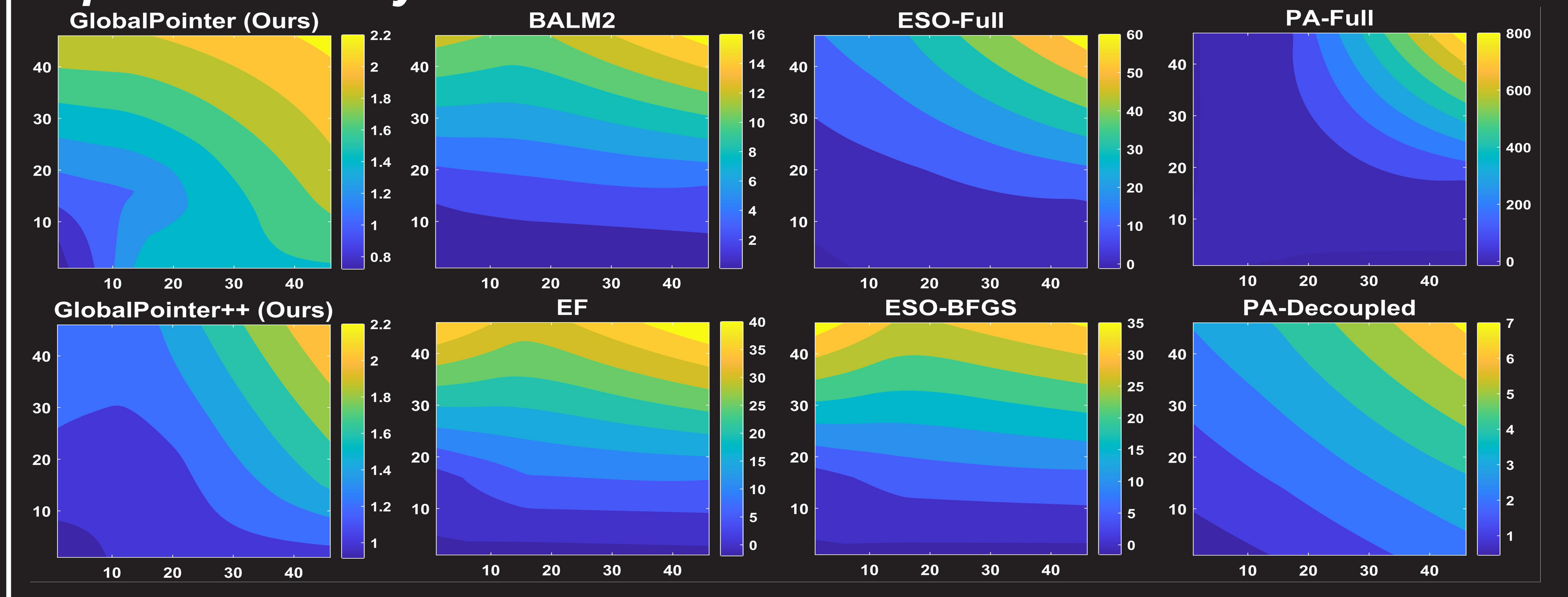
Pose-Only Closed-Form Solver → Plane-Only Closed-Form Solver → Pose-Only Closed-Form Solver

Experiments: Real Dataset

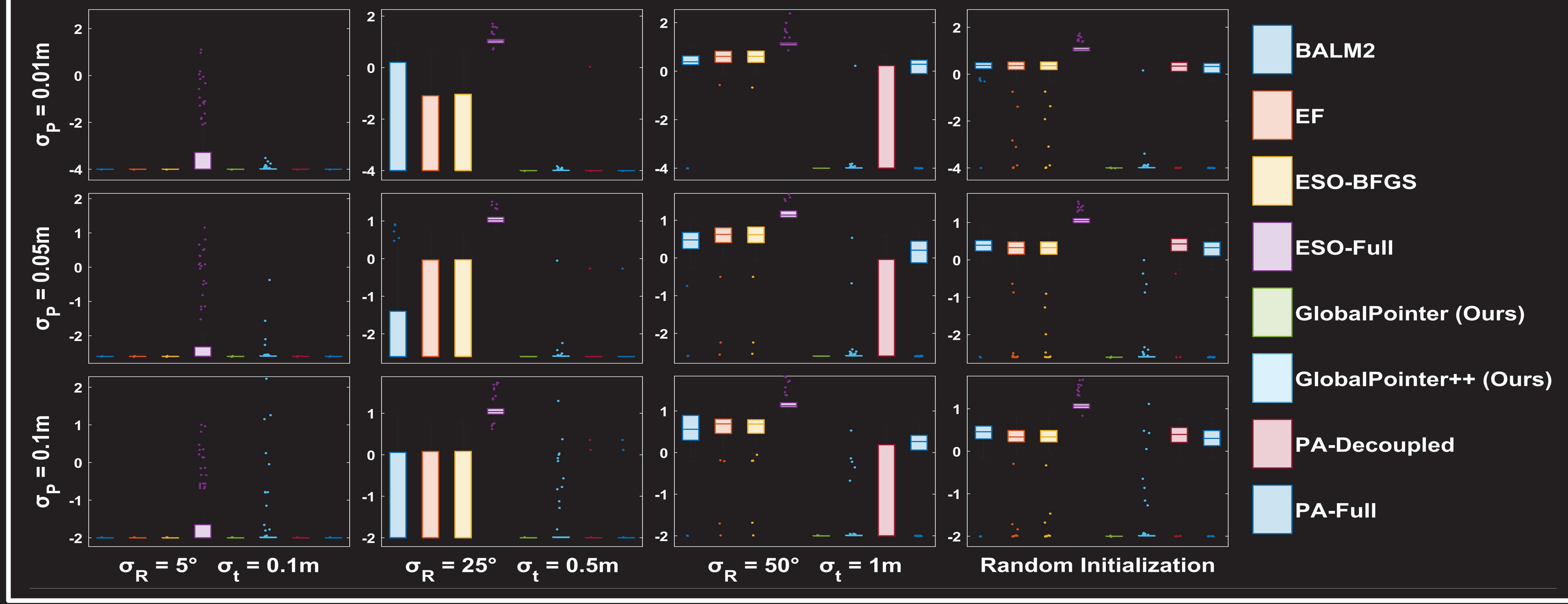


Accuracy comparisons on the real dataset under varying pose initialization noise levels.

Experiments: Synthetic Dataset



Relative time complexity comparisons against planes and poses. Time complexity is measured as the multiple of the optimization time relative to the initial setting.



Accuracy comparisons under varying point cloud noise levels and pose initialization noise levels.